

# The Impact of Magnolia's Mobile Transmitter Diversity Schemes on the Interference Cancellation Gains

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February 3, 2007

## Executive Summary

Let  $G_1$  be the capacity gain of a CDMA system when an interference cancellation algorithm is employed, and  $G_2$  denote the capacity gain of this system when mobile units employ any one of the Magnolia's transmit diversity techniques. If  $G$  denotes the capacity gain of the system when both the interference cancellation algorithm and Magnolia transmit diversity technique are employed, then it is proved that  $G = G_1G_2$ . Thus the capacity gains of interference cancellation techniques and Magnolia's mobile transmit diversity techniques are independent of each other.

# 1 Introduction

Magnolia's mobile transmitter diversity (MTD) method is an interesting technique that can produce diversity gain using the power control bit transmitted by the base-station, and by employing multiple transmit antenna chains at the mobile unit. The use of multiple transmit antennas at mobile terminal is limited by the physical size of the device, and for all practical cases, the number of mobile transmit antennas is limited to  $M = 2$ . At the base station, larger numbers of receive antennas are possible, with typical values of  $N = 1$  or  $N = 2$  receive antennas.

There are essentially two variations of Magnolia's method:

- **MTD-EGC**: Only the phases of the signals transmitted from the mobile antennas are adjusted to produce coherent combining of signals transmitted from mobile antennas at the base-station receiver. Clearly, when the base station has  $N = 1$  receive antenna, and assuming the convergence of the MTD algorithm to its stable point, the performance of the scheme is equivalent to two level equal gain diversity combining (EGC).
- **MTD-MRC**: Both phases and amplitudes of the signals transmitted from the mobile antennas are adjusted. In this case, under the assumption that the base station has  $N = 1$  receive antenna and assuming the convergence of the MTD algorithm to its stable point, the performance of the scheme is equivalent to two level maximum ratio diversity combining (MRC).

Recently, many discussions about interference cancellation techniques (ICT) are ongoing. In light of these, the author was asked to quantify the mutual impact of ICT and Magnolia's MTD techniques. In particular, Magnolia is interested in determining whether the deployment of ICT will decrease, increase or has no impact on the gains afforded by MTD. The purpose of the present document is to address these issues.

The outline of this report is given next. In Section 2, we analyze the impact of Magnolia's MTD schemes on  $P_{oc}$ , the out-of-cell, and  $P_{hc}$  home-cell power interference levels. We use this to quantify the effect of Magnolia's scheme on the F-factor of CDMA systems. We then compute the capacity gains of Magnolia's MTD scheme in the absence of interference

cancellation techniques (ICT). In Section 3, we will prove the main result of this paper. We will show that if

- $G_1$  denotes the capacity gain of a CDMA system, when an interference cancellation algorithm is employed,
- $G_2$  denotes the capacity gain of this system when Magnolia transmit diversity mobiles are employed, and
- $G$  denotes the capacity gain when both the interference cancellation algorithm and Magnolia transmit diversity mobiles are employed,

then  $G = G_1 G_2$ . It is concluded that the capacity gains of interference cancellation techniques and Magnolia's mobile transmit diversity techniques are independent of each other.

## 2 The Impact of MTD on F-factor

In order to compute the impact of Magnolia MTD on the F-factor, we first focus on an individual mobile unit. We will quantify the changes on the ratio of home cell to out of cell interference caused by this mobile with or without the Magnolia's MTD scheme.

### 2.1 Analyzing Interference Generated by a Mobile Without MTD

Consider a mobile unit  $u$  which has path gains  $PL_1(u)\gamma_h(u)$  and  $PL_2(u)\gamma_n(u)$  respectively to its home base-station antenna and a given neighboring base station, where  $PL_i(u)$ ,  $i = 1, 2$  denote the effect of long term fading and shadowing and  $\gamma_h(u)$  and  $\gamma_n(u)$  capture the effect of short term fading. If the power control loop of the home base station tries to maintain the received power of this mobile unit at the base station at  $P_{th}$  level, then we have

$$PL_1(u)^2 |\gamma_h(u)|^2 P_{tr}(u) = P_{th},$$

where  $P_{tr}(u)$  is the transmitted power out of the antenna of the mobile unit  $u$ . The out of cell interference caused by this mobile at the neighboring base-station receive antenna is given by

$$P_{oc,c}(u) = PL_2(u)^2 |\gamma_n(u)|^2 P_{tr} = \frac{PL_2(u)^2 |\gamma_n(u)|^2 P_{th}}{PL_1(u)^2 |\gamma_h(u)|^2}.$$

In other words

$$\frac{P_{oc,c}(u)}{P_{th}} = \frac{PL_2(u)^2 |\gamma_n(u)|^2}{PL_1(u)^2 |\gamma_h(u)|^2}.$$

## 2.2 Analyzing Interference Generated by a Mobile With MTD

Next consider the case that Magnolia's scheme is employed. In this case, the mobile has two transmit antennas. Let the path gain from these antennas (respectively antenna 1 and antenna 2) to the home and neighboring base-stations be respectively given by  $PL_1(u)(\alpha_h(u), \beta_h(u))$  and  $PL_2(u)(\alpha_n(u), \beta_n(u))$ .

### 2.2.1 MTD-MRC

We first consider the case that the mobile is performing maximum ratio combining transmit diversity. In this case, the mobile divides its total transmit power  $P_{tr}(u)$  between the two transmit antennas. Let us assume that a signal  $c$  is to be transmitted with power  $|c|^2 = P_{tr}(u)$ . If the Magnolia's MTD-MRC converges to the optimum stable value, then the signals  $\frac{\alpha_h(u)^*c}{\sqrt{|\alpha_h(u)|^2 + |\beta_h(u)|^2}}$  and  $\frac{\beta_h(u)^*c}{\sqrt{|\alpha_h(u)|^2 + |\beta_h(u)|^2}}$  are transmitted from antennas one and two respectively. This gives the signal  $PL_1(u)\sqrt{|\alpha_h(u)|^2 + |\beta_h(u)|^2}c$  at the base-station receive antenna, which has power  $PL_1(u)^2(|\alpha_h(u)|^2 + |\beta_h(u)|^2)P_{tr}(u)$ . As in the above, power control loop forces the value of received power to be  $P_{th}$ . This gives

$$PL_1(u)^2(|\alpha_h(u)|^2 + |\beta_h(u)|^2)P_{tr}(u) = P_{th}.$$

The out of cell interference in this case is given by

$$PL_2(u) \frac{(\alpha_h(u)^*\alpha_n(u) + \beta_h(u)^*\beta_n(u))c}{\sqrt{|\alpha_h(u)|^2 + |\beta_h(u)|^2}}$$

which gives out of cell power of

$$P_{oc,MRC}(u) = PL_2(u)^2 \frac{|\alpha_h(u)^*\alpha_n(u) + \beta_h(u)^*\beta_n(u)|^2 P_{tr}(u)}{|\alpha_h(u)|^2 + |\beta_h(u)|^2}.$$

In other words

$$\frac{P_{oc,MRC}(u)}{P_{th}} = \frac{PL_2(u)^2 |\alpha_h(u)^*\alpha_n(u) + \beta_h(u)^*\beta_n(u)|^2}{PL_1(u)^2 (|\alpha_h(u)|^2 + |\beta_h(u)|^2)^2}.$$

### 2.2.2 MTD-EGC

We next consider the case that the mobile is performing equal gain combining transmitter diversity. In this case, the mobile divides its total transmit power  $P_{tr}$  equally between the two transmit antennas. However, by using phase shifts, it compensates for the phases of the channels between mobile transmitter antenna and the receiver antenna at the base-station. Suppose that the signal  $c$  is to be transmitted with power  $|c|^2 = P_{tr}$ .

If the Magnolia's MTD-MRC converges to the optimum stable value, then the signals  $e^{-j\angle\alpha_h(u)}c/\sqrt{2}$  and  $e^{-j\angle\beta_h(u)}c/\sqrt{2}$  are respectively transmitted from antennas one and two of the mobile unit. This produces the signal  $(|\alpha_h(u)| + |\beta_h(u)|)c/\sqrt{2}$  at the base-station, which has power  $(|\alpha_h(u)| + |\beta_h(u)|)^2 P_{tr}/2$ .

As in the above, the power control loop forces the value of received power to be  $P_{th}(u)$ . This gives

$$PL_1(u)^2 \frac{(|\alpha_h(u)| + |\beta_h(u)|)^2 P_{tr}(u)}{2} = P_{th}.$$

The out of cell interference in this case is given by

$$PL_2(u) \frac{e^{j\angle\alpha_h(u)}\alpha_n(u) + e^{j\angle\beta_h(u)}\beta_n(u)}{\sqrt{2}}c$$

which gives out of cell power of

$$P_{oc,EGC} = \frac{PL_2(u)^2 |e^{-j\angle\alpha_h(u)}\alpha_n(u) + e^{-j\angle\beta_h(u)}\beta_n(u)|^2 P_{tr}(u)}{2}.$$

In other words

$$\frac{P_{oc,EGC}(u)}{P_{th}} = \frac{PL_2(u)^2 |e^{-j\angle\alpha_h(u)}\alpha_n(u) + e^{-j\angle\beta_h(u)}\beta_n(u)|^2}{PL_1(u)^2 (|\alpha_h(u)| + |\beta_h(u)|)^2}.$$

## 2.3 Interference Reduction Due to Magnolia's MTD

We next analyze the out of cell interference reduction due to Magnolia's MTD. Let us focus first on a specific mobile unit. From the above analysis, we can compute the fraction decrease in out of cell interference caused by this mobile unit following the application of Magnolia's scheme.

We assume that:

- The power control loop maintains a receive power of  $P_{th}$  for all mobile units, whether or not they use the Magnolia's MTD scheme (perfect power control).
- All sectors have  $K$  users, and  $K \gg 1$  is large.
- The users are distributed uniformly in each sector.
- The voice activity factor for all cells is equal to  $V_{ac}$ .

Under these assumptions, the amount of home cell interference for each cell is easily seen to be  $(K - 1)P_{th}V_{ac}$ . Thus the  $F$ -factor (the ratio of out of cell to home cell interference) without using the Magnolia's scheme is given by

$$F_c = \frac{\sum_u P_{oc,c}(u)}{(K - 1)P_{th}V_{ac}} = \frac{1}{(K - 1)V_{ac}} \sum_u \frac{PL_2(u)^2 |\gamma_n(u)|^2}{PL_1(u)^2 |\gamma_h(u)|^2}.$$

where  $u$  runs over all out of cell users.

When Magnolia's maximum ratio combining scheme is used, the  $F$  factor is given by

$$F_{MRC} = \frac{1}{(K - 1)V_{ac}} \sum_u \frac{PL_2(u)^2 |\alpha_h(u)^* \alpha_n(u) + \beta_h(u)^* \beta_n(u)|^2}{PL_1(u)^2 (|\alpha_h(u)|^2 + |\beta_h(u)|^2)^2}.$$

and when equal gain combining is used

$$F_{EGC} = \frac{1}{(K - 1)V_{ac}} \sum_u \frac{PL_2(u)^2 |e^{-j\angle\alpha_h(u)} \alpha_n(u) + e^{-j\angle\beta_h(u)} \beta_n(u)|^2}{PL_1(u)^2 (|\alpha_h(u)| + |\beta_h(u)|)^2}.$$

In order to be able to quantify the reductions in F-factor for the above, we need to make the following convenient assumption:

- **Symmetry Assumption:** Consider a Honeycomb arrangement of cells. Then the interferers are symmetrically distributed around each cell.

This assumption is intuitively appealing and simplifies the mathematical analysis. Under this assumption, for a given home cell, out of cell interferers could be now be put into groups  $U_i$ ,  $i = 1, 2, 3, \dots$  each consisting of 2 interferers  $U_i = \{u_{i,1}, u_{i,2}\}$  with similar distance profile from both cells. That is  $PL_1(u_{i,j}) = PL_1(U_i)$  and  $PL_2(u_{i,j}) = PL_2(U_i)$  independent of  $j = 1, 2$ .

We can then re-write

$$F_{MRC} = \frac{1}{(K-1)V_{ac}} \sum_{U_i} \frac{PL_2(U_i)^2}{PL_1(U_i)^2} \sum_{j=1}^2 \frac{|\alpha_h(u_{i,j})^* \alpha_n(u_{i,j}) + \beta_h(u_{i,j})^* \beta_n(u_{i,j})|^2}{(|\alpha_h(u_{i,j})|^2 + |\beta_h(u_{i,j})|^2)^2},$$

$$F_{EGC} = \frac{1}{(K-1)V_{ac}} \sum_{U_i} \frac{PL_2(U_i)^2}{PL_1(U_i)^2} \sum_{j=1}^2 \frac{|e^{-j\angle\alpha_h(u_{i,j})} \alpha_n(u_{i,j}) + e^{-j\angle\beta_h(u_{i,j})} \beta_n(u_{i,j})|^2}{(|\alpha_h(u_{i,j})| + |\beta_h(u_{i,j})|)^2},$$

and

$$F_c = \frac{1}{(K-1)V_{ac}} \sum_{U_i} \frac{PL_2(U_i)^2}{PL_1(U_i)^2} \sum_{j=1}^2 \frac{|\gamma_n(u_{i,j})|^2}{|\gamma_h(u_{i,j})|^2}.$$

Since  $K$  is large, so is the number of interferers. Let the number of interferers be  $2L$ , then the number of sets  $U_i$  is  $L$ . We next apply the weak law of large numbers, and write

$$F_{MRC} = \frac{L}{(K-1)V_{ac}} \mathbf{E} \left( \frac{PL_2(U)^2}{PL_1(U)^2} \sum_{j=1}^2 \frac{|\alpha_h(u_j)^* \alpha_n(u_j) + \beta_h(u_j)^* \beta_n(u_j)|^2}{(|\alpha_h(u_j)|^2 + |\beta_h(u_j)|^2)^2} \right),$$

$$F_{EGC} = \frac{L}{(K-1)V_{ac}} \mathbf{E} \left( \frac{PL_2(U)^2}{PL_1(U)^2} \sum_{j=1}^2 \frac{|e^{-j\angle\alpha_h(u_j)} \alpha_n(u_j) + e^{-j\angle\beta_h(u_j)} \beta_n(u_j)|^2}{(|\alpha_h(u_j)| + |\beta_h(u_j)|)^2} \right),$$

and

$$F_c = \frac{L}{(K-1)V_{ac}} \mathbf{E} \left( \frac{PL_2(U)^2}{PL_1(U)^2} \sum_{j=1}^2 \frac{|\gamma_n(u_j)|^2}{|\gamma_h(u_j)|^2} \right),$$

where  $\mathbf{E}(\cdot)$  denote the expectation operator, and  $U$  is a set of 2 symmetric interferers  $U = \{u_1, u_2\}$  distributed uniformly in a cell.

Since the long term fading and shadowing are independent of short term fading, we can write

$$\begin{aligned} & \mathbf{E} \left( \frac{PL_2(U)^2}{PL_1(U)^2} \sum_{j=1}^2 \frac{|\alpha_h(u_j)^* \alpha_n(u_j) + \beta_h(u_j)^* \beta_n(u_j)|^2}{(|\alpha_h(u_j)|^2 + |\beta_h(u_j)|^2)^2} \right) \\ &= \mathbf{E} \left( \frac{PL_2(U)^2}{PL_1(U)^2} \right) \mathbf{E} \left( \sum_{j=1}^2 \frac{|\alpha_h(u_j)^* \alpha_n(u_j) + \beta_h(u_j)^* \beta_n(u_j)|^2}{(|\alpha_h(u_j)|^2 + |\beta_h(u_j)|^2)^2} \right), \end{aligned}$$

$$\begin{aligned}
& \mathbf{E}\left(\frac{PL_2(U)^2}{PL_1(U)^2} \sum_{j=1}^2 \frac{|e^{-j\angle\alpha_h(u_j)}\alpha_n(u_j) + e^{-j\angle\beta_h(u_j)}\beta_n(u_j)|^2}{(|\alpha_h(u_j)| + |\beta_h(u_j)|)^2}\right) \\
&= \mathbf{E}\left(\frac{PL_2(U)^2}{PL_1(U)^2}\right)\mathbf{E}\left(\sum_{j=1}^2 \frac{|e^{-j\angle\alpha_h(u_j)}\alpha_n(u_j) + e^{-j\angle\beta_h(u_j)}\beta_n(u_j)|^2}{(|\alpha_h(u_j)| + |\beta_h(u_j)|)^2}\right),
\end{aligned}$$

and

$$\mathbf{E}\left(\frac{PL_2(U)^2}{PL_1(U)^2} \sum_{j=1}^2 \frac{|\gamma_n(u_j)|^2}{|\gamma_h(u_j)|^2}\right) = \mathbf{E}\left(\frac{PL_2(U)^2}{PL_1(U)^2}\right)\mathbf{E}\left(\sum_{j=1}^2 \frac{|\gamma_n(u_j)|^2}{|\gamma_h(u_j)|^2}\right).$$

We can now state the following theorem.

**Theorem 1** *Let  $F_c$ ,  $F_{MRC}$  and  $F_{EGC}$  denote the  $F$  factors with no Magnolia scheme, Magnolia's maximum ratio combining transmitter diversity and Magnolia's Equal gain combining. Then*

$$\frac{F_{MRC}}{F_c} = \frac{\mathbf{E}\left(\sum_{j=1}^2 \frac{|\alpha_h(u_j)^*\alpha_n(u_j) + \beta_h(u_j)^*\beta_n(u_j)|^2}{(|\alpha_h(u_j)|^2 + |\beta_h(u_j)|^2)^2}\right)}{\mathbf{E}\left(\sum_{j=1}^2 \frac{|\gamma_n(u_j)|^2}{|\gamma_h(u_j)|^2}\right)}, \quad (1)$$

$$\frac{F_{EGC}}{F_c} = \frac{\mathbf{E}\left(\sum_{j=1}^2 \frac{|e^{-j\angle\alpha_h(u_j)}\alpha_n(u_j) + e^{-j\angle\beta_h(u_j)}\beta_n(u_j)|^2}{(|\alpha_h(u_j)| + |\beta_h(u_j)|)^2}\right)}{\mathbf{E}\left(\sum_{j=1}^2 \frac{|\gamma_n(u_j)|^2}{|\gamma_h(u_j)|^2}\right)}, \quad (2)$$

where  $\alpha(u_j), \beta(u_j), \gamma(u_j)$ ,  $j = 1, 2$  are short term fading coefficients.

**Proof:** The proof easily follows by combining the above equalities.  $\square$

## 2.4 Magnolia's MTD Capacity Increase

Using the above theorem, we compute the ratios  $\frac{F_{MRC}}{F_c}$  and  $\frac{F_{EGC}}{F_c}$  under the assumption that for each  $j = 1, 2$ :

- $\gamma_h(u_j) = \alpha_h(u_j)$  and  $\gamma_n(u_j) = \alpha_n(u_j)$  (This assumes that the first transmit antenna for a Magnolia's mobile unit is co-located with that of a classical mobile unit that does not use Magnolia's scheme).
- $\beta_h(u_j)$  and  $\beta_n(u_j)$  have respectively correlation  $r = 0.5$  with  $\alpha_h(u_j)$  and  $\alpha_n(u_j)$ ,
- The pair  $\alpha_h(u_j)$ , and  $\alpha_n(u_j)$  are independent, and

$F_{MRC}/F_c$	$F_{EGC}/F_c$	$C_{MRC}/C_c$	$C_{EGC}/C_c$
0.3620	0.4068	1.4147	1.3746

Table 1: F-Factor Reductions and Capacity Gains for Magnolia Schemes for Antenna Correlation Factor = 0.5

- The pair  $\beta_h(u_j)$ , and  $\beta_n(u_j)$  are independent.
- The difference between the orientation of mobile units with various base-stations are taken into account.

The results are presented in Table 1.

For a CDMA system, it is well known that number of users in the system is given by the Pole Capacity Formula

$$\text{Number of Users} = \frac{1}{(1 + F)V_{ac}\text{SINR}} + 1 \simeq \frac{1}{(1 + F)V_{ac}\text{SINR}} \quad (3)$$

where  $V_{ac}$  is the voice activity ratio, and SINR is the signal to interference and noise ratio for each user. From Magnolia's experience, the typical value of  $F$  is about 0.85 (without the application of Magnolia scheme). Using this value and the above formula, we can compute approximate values of the ratios of capacities for Magnolia's MTD-MRC and MTD-EGC schemes over the capacity with no diversity scheme for antenna correlation factor  $r = 0.5$ . These are respectively denoted by  $C_{MRC}/C_c$  and  $C_{EGC}/C_c$ , and are tabulated in Table 1. From the above, one expects to observe capacity gains of about 37% to 41% using the Magnolia transmit diversity schemes.

### 3 Magnolia's MTD Gain in The Presence of ICT

In this section, we will show that the gains of Magnolia MTD schemes are independent of those of ICT. In order to prove this result, we make the following assumptions:

#### ICT Assumptions:

- The base-station receiver does not have any knowledge of the channels of out-of-cell interferers.

- The base-station receiver treats the out-of-cell interference as Gaussian noise.
- The ICT algorithm uses linear combining of signals received at receive antennas  $1, 2, \dots, N$  of the base-station.
- All the underlying sub-channels from each given uplink user to receive antennas of the base-station are i.i.d. Gaussian with zero mean.
- If the ICT algorithm is applied, then it is applied in every cell.
- The short term fading components of the channels of all uplink users to the base-station receive antennas are all i.i.d. Gaussian with zero mean.

The above assumptions are relatively realistic, since each base-station usually only un.masks the long code of it's home cell users. Also, in typical applications, only linear interference cancellation techniques (such as MSINR or zero-forcing interference cancellation techniques) are used.

Let us assume now that ICT is employed in cell  $A$ . Let the signal vector received at receive antenna  $j = 1, 2, \dots, N$  at time  $t$  be  $r_j(t)$ . Let  $\mathbf{r}(t) = (r_1(t), \dots, r_N(t))^T$ , where the column vector  $(r_1(t), \dots, r_N(t))^T$  is the transpose of the row vector  $(r_1(t), \dots, r_N(t))$ .

Let  $H_c$  be an  $N \times K$  matrix corresponding to the channels between home cell uplink users  $1, 2, \dots, K$  to receive antennas  $1, 2, \dots, N$ , assuming that Magnolia scheme is not used in cell  $A$ . For the case, when Magnolia's mobile units are employed in cell  $A$ , let  $H_M$  denote the equivalent  $N \times K$  matrix corresponding to the combined channels between antennas 1 and 2 of each home cell uplink users  $1, 2, \dots, K$  to receive antennas  $1, 2, \dots, N$ . Let  $S = 2L$  denote the number of out of cell interferers. Similarly, let  $G_c$  and  $G_M$  respectively denote  $N \times S$  channel matrices of interfering signals for the classical and Magnolia mobiles. If the column vector of signals transmitted from antennas of home cell and out of cell users are  $\mathbf{c}(t)$  and  $\mathbf{c}_I(t)$ , (respectively  $\mathbf{c}_{Mag}(t)$  and  $\mathbf{c}_{Mag,I}(t)$ ), then

$$\mathbf{r}(t) = H_c \mathbf{c}(t) + G_c \mathbf{c}_I(t) + n_1(t) \quad (4)$$

for the classical mobile unit case. When Magnolia's mobile units are used, we have

$$\mathbf{r}(t) = H_M \mathbf{c}_{Mag}(t) + G_M \mathbf{c}_{Mag,I}(t) + n_2(t), \quad (5)$$

where  $n_1(t)$  and  $n_2(t)$  are Gaussian noise terms representing thermal noise with the same temperature.

We can now prove the following Lemma.

**Lemma 1** *Linear combining based ICT at Cell A does not decrease the power of out of cell interfering signals.*

**Proof:** For any uplink transmitter  $j = 1, 2, \dots, K$ , an ICT algorithm based on linear combining computes a vector  $\Lambda_j = (\lambda_{j,1}, \dots, \lambda_{j,N})$ , given the matrix  $H_c$  (respectively  $H_M$ ). It then forms the combined signal

$$\Lambda_j \mathbf{r}(t) = \Lambda_j H_c \mathbf{c}(t) + \Lambda_j G_c \mathbf{c}_I(t) + \Lambda_j n_1(t) \quad (6)$$

for the classical mobile unit case. When Magnolia's mobile units are employed, the combined signal is given by

$$\Lambda_j \mathbf{r}(t) = \Lambda_j H_M \mathbf{c}_{Mag}(t) + \Lambda_j G_M \mathbf{c}_{Mag,I}(t) + \Lambda_j n_2(t). \quad (7)$$

Since,  $G_c$  (respectively  $G_M$ ) are not known, the vector  $\Lambda_j$  is only a function of  $H_c$  (respectively  $H_M$ ). Since the columns of  $G_c$  (respectively  $G_M$ ) are i.i.d Gaussian with zero mean, the statistics of columns of  $\Lambda G_c \mathbf{c}$  (respectively  $\Lambda G_M$ ) and  $\Lambda n_1(t)$  (respectively  $\Lambda n_2(t)$ ) are identical to those of  $G_c$  and  $n_1(t)$  (respectively  $G_m$  and  $n_2(t)$ ). This implies that the total power of out of cell interfering signals remains the same.  $\square$

In order to present the main result of this paper, we need to compute the effect of ICT gains on the pole capacity formula. To this end, consider a cell with  $K - 1$  home cell interfering uplink users and  $S = 2L$  interfering uplink users. The signals of all home cell users arrive at the base station with power  $P_{th}$ . Thus the home cell interference is given by  $P_{hc} = (K - 1)P_{th}V_{ac}$ , where  $V_{ac}$  is the activity factor. Let the out of cell interference power be  $P_{oc}$  and the thermal noise power be  $\sigma_T^2$ . Then  $SINR_c$  of the desired uplink user without the application of the ICT algorithm is given by

$$SINR_c = \frac{P_{th}}{(K - 1)P_{th}V_{ac} + P_{oc} + \sigma^2} \quad (8)$$

Now let us assume that the ICT algorithm is applied. The ICT algorithm produces an SINR gain of  $G$ . This means that  $SINR_{ICT}$  of the desired user following the application of the ICT is given by

$$SINR_{ICT} = \frac{GP_{th}}{(K-1)P_{th}V_{ac} + P_{oc} + \sigma^2} \quad (9)$$

By previous lemma, the ICT algorithm does not affect the out of cell interference power  $P_{oc}$  and  $\sigma^2$ . Thus the  $F$  factor is still given by

$$F = \frac{P_{oc}}{(K-1)P_{th}V_{ac}}. \quad (10)$$

Assuming that the system is heavily loaded, we can ignore the contribution of the thermal noise  $\sigma^2$  in Equation (9) and solve for  $K$  to have

$$K = \frac{G}{SINR_{ICT}V_{ac}(1+F)}, \quad (11)$$

where  $SINR_{ICT}$  is the SINR of uplink users after the application of the ICT algorithm.

We can now prove the following Theorem:

**Theorem 2** *The Capacity Gain of Magnolia's Schemes are independent of those of interference cancellation algorithms.*

**Proof:** Consider a cell with  $K$  home cell users and with power control loop target  $P_{th}$ . It is seen from the capacity formula given in Equation (11) that

$$K = \frac{G}{SINR_{ICT}V_{ac}(1+F)}, \quad (12)$$

where  $SINR_{ICT}$  is the target receive SINR of the uplink users (after the application of ICT).

Comparing this with the classical pole capacity equation, we see that the capacity gain of the ICT algorithm is  $G$ . The capacity gains of the Magnolia schemes are obtained by affecting the  $F$  in the above, which is clearly independent of  $G$ .  $\square$

## **About The Author**

Vahid Tarokh received the PhD in Electrical Engineering from the University of Waterloo, Ontario, Canada in 1995. He then worked at AT&T Labs-Research until August 2000, where he was the head of the Department of Wireless Communications and Signal Processing. In September 2000, he joined Department of Electrical Engineering and Computer Sciences (EECS) at MIT as an associate professor. In June 2002, he joined Harvard University as a Gordon McKay Professor of Electrical Engineering . Since July 2005, he is a Hammond Vinton Hayes Senior Fellow of Electrical Engineering at Harvard University, and Perkins Professor of Applied Mathematics. His research is mainly focused in the areas of Signal processing, Communications (wireline and wireless) and Networking .

Dr. Tarokh is one of the Top 10 Most Cited Researchers in Computer Science according to the ISI web of science. He has received a number of awards and holds 2 honorary degrees.